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Direction of Arrival Estimation using MUSIC Algorithm

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Abstract— *The research in DOA technology is an important area in array signal processing. DOA estimation of coherent signals, multiple signals and broadband signals are of greater consideration .When an incoming wave is received by an array or a set of arrays, the received wave can be used and processed to derive many related information of the associated wave .The Direction of arrival angle is one such estimation. This estimation has vast applications in sonar, radar, seismology, astronomy, earthquake, communication and biomedicine. The conventional MUSIC (Multiple Signal Classification) method of Direction of arrival estimation uses uniform linear arrays in which the array elements are spaced in such a manner that they meet the Nyquist sampling criteria. The accuracy in the estimation of direction of arrival is very crucial in array signal processing. This paper presents an overview of the MUSIC algorithm being used for direction of arrival estimation and the factors that can affect the DOA estimation results. In this paper I will be describing the concept regarding the estimation of arrival angle and provide some MATLAB simulations to highlight the factors that can improve accuracy. Attempts are made to bring about better resolution and contrast. The algorithm here mainly focuses on the use of radio frequencies, but this can even be extended for acoustic and mechanical waves.*

1. INTRODUCTION

The effectiveness of the direction of arrival estimation algorithm greatly determines the performance of the antenna. In signal processing the objective is always to measure a set of constant parameters upon which the received signal depends. The received signal impinging on the array consists of a bunch of important parameters out of which the angle with which the signal is received is very vital for estimating the location .The aim of array signal processing is to process the signals that are received by the sensor array and then strengthen the useful signals by restraining noise and interference. Applications of array signal processing (ASP) include sonar, biomedicine, astronomy, radar, wireless communication system [1], prediction of seismic events, radar etc. The arrival angle can be estimated either using the classical or else the subspace based method. The overall spatial spectrum can be thought to be an assembly of three stages namely the target, observation and the estimation stage[18] .There are many algorithms like ESPRIT, MUSIC, WSF, MVDR, ML techniques [4] and others which can be used for the estimation process. In maximum Likelihood (ML) estimation a parameter space of higher dimension is to be searched, which requires a vast number of calculations. Hence it is difficult to put this estimation method into practice [5, 6].

The Burg's method proposed in 1967 includes Auto Regressive (AR) model, Maximum Entropy model, Moving Average (MA) Model, the Auto Regressive and Moving Average (ARMA) Model. These models have low robustness [5,6] and require abundant calculations. ESPRIT (Estimation of Signal Parameter via Rotational Invariance Technique) and MUSIC(Multiple Signal Classification) are spectral estimation methods which deal with the decomposition of Eigen values. ESPRIT has relatively narrow applications as it is restricted to be used only with some peculiar array structure [7]. Among these algorithms the MUSIC algorithm is the most classic one, which can be used for both uniform and non-uniform linear arrays. The subspace based approaches were found to depend on the covariance matrices of the signals. This lead to an extensive use of the subspace based approaches like ESPRIT [15] and MUSIC [2,3] algorithms in the estimation of the arrival angle. MUSIC (Multiple Signal Classification) is the most accepted parameter estimation algorithm. It can compute the number of signals that are being incident on the sensor array, the strength of these signals and the direction i.e. the angle from which the signal are being incident. In this DOA estimation firstly the bearing space is sampled uniformly to get many discrete angles. Then we assume that the source and noise signals arrive from every small bearing angles and the estimation algorithm computes the angle of signal with stronger power. The estimation process can even be brought about by simply measuring the phase difference of two array elements[17] that are equivalent, but this requires that the phase angle indicator to be too accurate and work flawlessly because a small error in even one of the indicator will lead to large deviation in the final estimated angle. The resolution of the sonar array greatly depends on the aperture of the array, so in order to improve the estimation process for the underwater targets, the aperture of the sonar array needs to be increased.

The non-uniform array [8, 9] can be designed by using various tools but it has various limitations like it can be applied only to those situations where the source signal is completely independent of the noise and only when the array consists of fewer sensor elements. It is quite possible to achieve such non-noise conditions in air but when it comes to the underwater environment which is much denser and complex, achieving such conditions is not feasible. In order to bring about better estimation results in such complex scenario the concept of random arrays [12] is implemented. This method results in the dispersing of the tall side lobes and the targets which are centralize are prominently detected. The random arrays can even provide better results by compensating the signal subspace when the signals are coherent. The conventional MUSIC method uses the concept of uniform linear arrays i.e. the array spacing between the consecutive elements remains the same throughout.

2. MATHEMATICAL MODEL FOR DIRECTION OF ARRIVAL

Considering that there are M source signals that have the same center frequency f_0 and each of them are narrow band signals impinging on a array which has D elements. The D elements here are equally spaced, and the number is always greater than the number of impinging signals i.e.(D>M) .We assume that these signals impinge on the sensor array with azimuth angles θ_k , 1≤k ≤M.

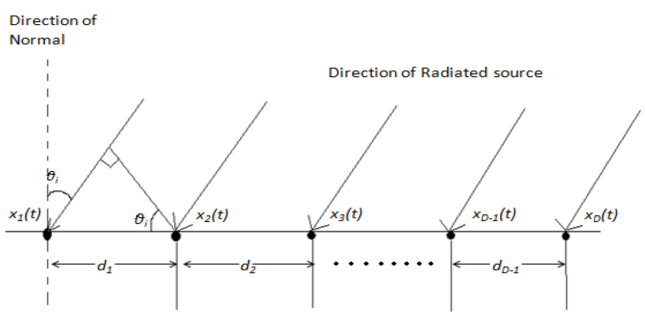

Fig. 1: Structure of Sonar Array

It is assumed that the signals and the array are in the same plane. As the elements are equally spaced so the distance between consecutive elements is same and can be represented as a space matrix [d_1, , d_2,…d_{D-1}],where d is always smaller than half the wavelength of the signal with the highest frequency [10] and $d_1=d_2= \ldots=d_{D-1}$. As the source signal is supposed to be a narrowband far- field signal it can be represented in its complex form as :

$$S(t)= a(t)\, e^{j(f_0(t)+\phi(t))} \quad (1)$$

where $\phi(t)$ is assumed to be the phase of the source signal, and *a(t)* is the amplitude . Now considering the first element of the array as reference, the signal that will be sensed by the k^{th} source at the p^{th} array is represented as

$$S_k(t)\, e^{\frac{-j2\pi d_p (p-1)\sin\theta_k}{\lambda}} \text{ where } 1\le k \le M \quad (2)$$

Now for all sources , the sensing at the p^{th} array would be:

$$b_p(t) = \sum_{k=1}^{M} e^{\frac{-j2\pi d_p (p-1)\sin\theta_k}{\lambda}} s_k(t) + n_p(t) \quad (3)$$

where 1≤ p ≤D.

d_p is the distance between the consecutive $(p-1)^{th}$ and the p^{th} elements of the array , S_k *(t)* represents the k^{th} source signal, n_p is the noise signal sensed at the p^{th} array . The entire signal information received by the M signal sources and the D array elements can be segregated using (2) and (3) to a compressed form as shown here :

$$B(t) = AS(t) + N(t) \quad (4)$$

where A is called the steering vector matrix, N(t) represents the noise . B(t) is the received signal in a matrix form and can be given as:

$$B(t) = [\, b_1(t)\; b_2(t) \ldots b_D(t)]^T \quad (5)$$

and the source matrix is given by

$$S(t) = [\, s_1(t) ,\, s_2(t), \ldots s_M(t)\,]^T \quad (6)$$

The steering vector matrix [11] is represented as:

$$A= [\, \Upsilon(\theta_1) ,\, \Upsilon(\theta_2), \ldots, \Upsilon(\theta_M)\,] \quad (7)$$

$$\Upsilon(\theta_k)= [1\; e^{-j\frac{2\pi d_1}{\lambda}\sin(\theta k)} \ldots e^{-j(D-1)\frac{2\pi d_{D-1}}{\lambda}\sin(\theta k)}\,]^T$$

and 1≤ k ≤M.

Now this can even be extended to evaluate the conditions for random signal array if we assume that the source signal hits the sonar based array with an angle θ_1 , then the phase difference on considering 'd_k' to be the random spacing and

1 ≤ k ≤ D-1.

$$\Phi_k(\theta_1) * \lambda \mathrm{Sin}(\theta_k) = 2\pi d_k \quad (8)$$

Now considering the situation with an ambiguous phase 'θa' while estimating the sources i.e. θa is not equal to θ_1, this inequality leads to the following equation:

$$\Phi_i(\theta_a) = \Phi_i(\theta_1) + 2\pi k_k \quad (9)$$

where 1 ≤ i≤ D-1.

As long as the array element spacing is random, we can never find the value for k_i to satisfy the steering vector matrix, i.e. θ_a not equal to θ_1 is never true. Hence $\theta_a = \theta_1$. This relation proves that there is very less probability of having ambiguous angle detection.

3. PRINCIPLE OF MUSIC ALGORITHM

Schmidt and his colleagues proposed this algorithm in 1979[14] . The MUSIC algorithm uses the eigen value decomposition of the covariance matrix. Although a various estimation algorithms can be used to compute the angle of arrival, but this paper focuses on the most accepted and widely used MUSIC algorithm. The data correlation matrix forms the base of MUSIC algorithm. The steering vector as discussed earlier in the random data model correspond to the incoming signals and hence forms the signal subspace. This signal subspace is exactly orthogonal to the noise subspace and this form of orthogonality between these two subspaces can be exploited to measure the arrival angle. To find the direction of arrival we need to search through the entire steering vector matrix and then bring out those steering vectors that are exactly orthogonal.

The covariance matrix R_B, for output B is given as

$$R_B = E[BB^H] \quad (10)$$

Now using (3)

$$R_B = E[(AS+N)(AS+N)^H]$$
$$= AE[SS^H]A^H + E[NN^H]$$
$$= AR_SA^H + R_N \quad (11)$$

where R_N is the signal correlation matrix , A is the steering vector matrix and R_N is the noise correlation matrix .

$$R_N = \sigma^2 T \quad (12)$$

where T is unit matrix of D*D. Considering the ideal case, i.e. the situation with no noise

$$R_B = AR_SA^H \quad (13)$$

The rank of R_B is equal to the number of signal sources only when R_B is a non-singular matrix, i.e. when each column of the steering vector matrix are independent. In presence of noise

$$R_B = AR_SA^H + R_N \quad (14)$$

since $\sigma^2 > 0$, R_B will be a full rank matrix with positive real eigen values $\lambda_1, \lambda_2, \ldots, \lambda_D$ .The M eigen values are related to the source signal while the remaining D-M corresponds to noise. Now R_B can be decomposed as : $R_B = U_S \sum U_S^H + U_N \sum U_N^H$, where U_s and U_N represents the signal subspace and noise subspace respectively. The larger eigen values form the signal subspace and the smaller eigen values form the noise subspace. Ideally the signal subspace is orthogonal to the noise subspace and hence the steering vectors are also orthogonal i.e.

$$\Upsilon^H(\theta)U_N = 0 \quad (15)$$

As the existence of noise, does not allow the signal subspace to be completely orthogonal to the noise subspace, the estimated arrival angle can be represented as :

$$\theta_{MUSIC} = \text{argmin} . \Upsilon^H(\theta)U_NU_N^H \Upsilon(\theta) \quad (16)$$

In order to obtain peaks, the relation can be represented as

$$P_{MUSIC} = \frac{1}{\Upsilon^H(\theta)U_NU_N^H\Upsilon(\theta)} \quad (17)$$

When the value of θ is equal to that of the arrival angle, this function will have very high values since the denominator of the equation approaches zero. Hence this process is a complete peak search method, where the high peaks of the spectrum corresponds to the arrival angles of the source signals. It basically estimates 'U_N' i.e. the basis for the noise subspace and then when associated with the arrival angle, it results in M peaks corresponding to the M signal sources. It produces maximum power for the direction that is to be estimated[16].

Considering the element spacing to be λ/2, number of array elements to be 20, Signal to Noise ratio (SNR) to be 10dB and number of snapshots to be 300. The simulation shown below represents the estimation result for two non-correlated narrow band signals with angle of incidence 30° and 70° respectively.

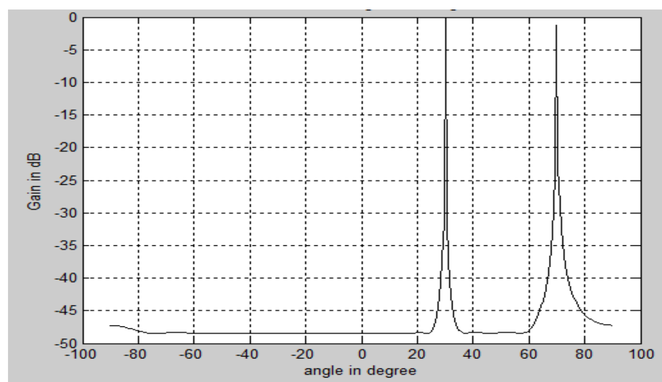

Fig. 2: Simulation result for basic MUSIC algorithm

The simulation result shows two independent spectrum peaks which can be used to estimate the direction with which the signals are incident and also estimates the number of source signals.

4. FACTORS AFFECTING DOA ESTIMATION RESULTS

The efficiency of estimation result is not only dependent on the incoming signal source, but also on the sensor array design and the application environment [13]. The following parameters can bring about variations in the efficiency outcome.

4.1 The number of Array Elements:

The performance of estimation process can be changed by varying the number of array elements in the array structure. It is seen that the performance can be improved by increasing the number of array elements. Considering two narrow band signals with SNR 10 dB which are independent of each other and are incident on the array of sensors with angles 30° and 70°. The number of snap shot is taken to be 300 and the noise here is the ideal Gaussian noise. The following simulation evaluates the relationship between the performance of MUSIC algorithm and the number of array elements.

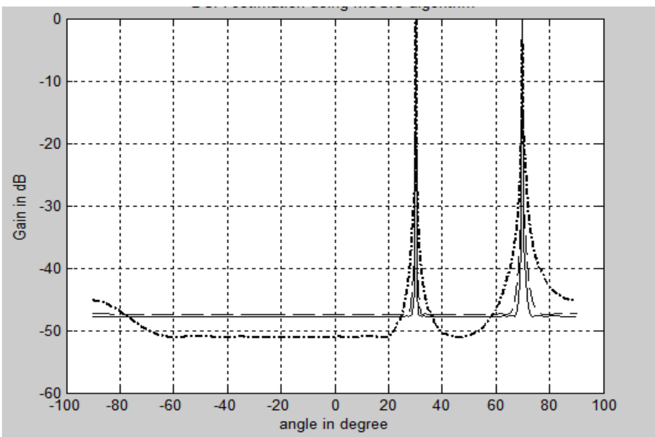

Fig. 3: Simulation result on varying the number of array elements

It can be seen that an increase in the number of array elements enhances the directivity and hence the signals are distinguished. Simulation corresponding to 10 element is represented by the dash-dotted line, the dashed line for 50 array elements and the simulation result for 100 element is given by the solid line. So the number of array elements can be increased to improve the accuracy of estimation. But this increase in the number of array elements also increases the computation time as the amount of data to be processed increases. Hence an appropriate selection is to be made by considering the trade-off between the computation time and the number of array elements.

4.2 Array element spacing

The performance can be changed by changing the spacing between the array elements. Initially all the array elements were supposed to be spaced at a distance of λ/6, then eventually the distance was changed to λ/2 and λ to see the varying simulation results. With all the conditions remaining the same, considering the noise to be an ideal Gaussian noise and with 10 number of array elements, the simulation result obtained as shown below

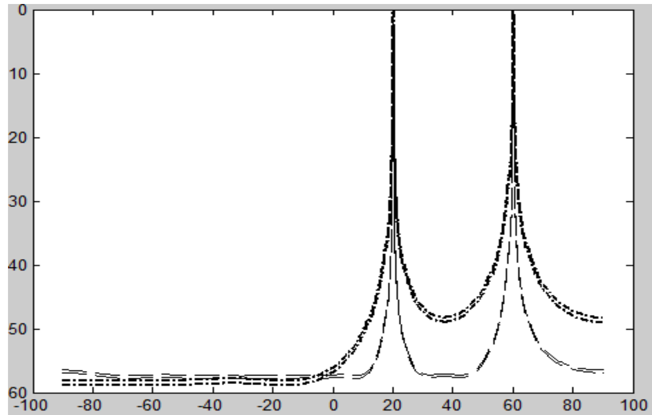

Fig. 4: Simulation result on varying the array element spacing

The simulation result for a spacing of λ/6 is represented by the dash-dotted line, simulations for λ/2 is shown by the dashed lines and the solid lines corresponds to the simulation result for a array spacing of length λ. It can be seen that MUSIC algorithm performs well only when the spacing between array elements is less than λ/2. If the array spacing is restricted to be λ/2, it is seen that with an increase in the element spacing the spectrum becomes narrower. Hence the resolution of MUSIC algorithm increases with an increase in array spacing if the maximum distance is maintained to be less than or equal to λ/2. The algorithm losses estimation accuracy and shows false peaks when the array element spacing gets larger than half of the wavelength. Array elements spaced with a distance of λ/2 give the best results.

4.3 Number of Snapshots

Keeping all the considerations same as in the basic simulation for MUSIC algorithm and changing the number of snapshots, we find that the performance of the estimation algorithm varies with the number of snapshots taken into account. The simulation below considers the number of array elements to be 10, and the number of snapshots to vary as 50, 100 and 200. The results for 50 snapshots are shown by the dash-dotted lines, simulation for 100 number of snapshot is represented by the dashed line and the solid line is for 200 snapshots. It can be seen that an increase in the number of snapshots makes the spectrum of estimation process much narrower. This narrower spectrum has higher directivity and accuracy. Hence the resolution and accuracy of the estimation process can be improved by using a higher number of snapshots.

The drawback of excessively expanding the number of snapshots is that it will increase the number of calculations and computations, which will further lower the speed of the estimation algorithm. Hence a selection of the number of snapshots is to be made appropriately by considering the trade-off between the computational complexity and accuracy.

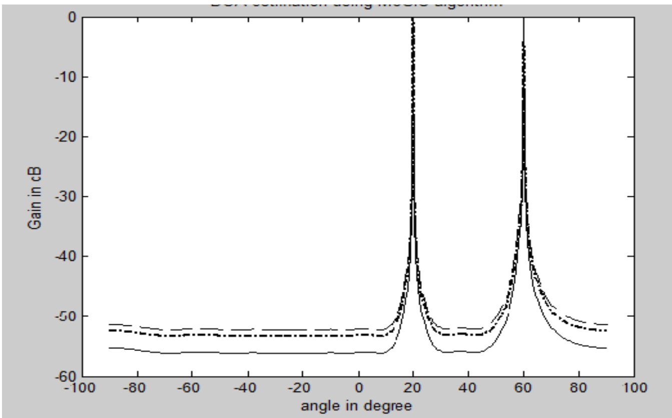

Fig. 5: Simulation result on varying the number of snapshots

5. CONCLUSION

MUSIC algorithm is an efficient spatial spectral estimation algorithm. The idea here is to carry out the decomposition of the data covariance matrix. This results in the formation of two subspaces namely the signal and the noise subspace, which are orthogonal to each other. These orthogonal subspaces can be exploited to estimate the direction of arrival angle. It can be concluded that the resolution and accuracy of MUSIC algorithm can be improved by increasing the number of array elements, by increasing the number of snapshots and even by increasing the inter element spacing provided that the maximum spacing is limited to a length of λ/2. We also find that false peaks are obtained when the inter element spacing is greater than λ/2.

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